Paper / Subject Code: 40921 / Engineering Mathematics-IV

Max. Marks: 80 **Time: 3 hrs.**

N.B. : 1. Q1 is compulsory
2. Attempt any three questions from Q2 to Q6.
3. Figures to the right indicate full marks.

Q1. (a) A r.v. X has the distribution [5]

X :	0	1	2	3	4	5	6
$p(x)$:	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Find i) k ii) $P(3 < X \le 6)$.

(b) Evaluate the integral $\int_C \frac{z^2}{(z-3)^2 (z+2)} dz$, C: $|z+1| = 2$. [5]

(c) Using Gram Schmidt method, find an orthonormal set of vectors corresponding to $\{(3,0,4), (1,0,2)\}$. [5]

(d) The given data indicates weight x and heights y of 1000 men. $\bar{x} = 150$ lbs, $\bar{y} = 68$ inches, $\sigma_x = 20$ lbs, $\sigma_y = 2.5$ inches, $r = 0.6$. Find the line of regression of y on x and estimate the height of a person whose weight is 200 lbs. [5]

Q2. (a) If $f(x) = \begin{cases} \frac{x}{2} & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$ is a pdf of a random variable X, then find $E(X)$, $var(X)$, $var(3X)$. [6]

(b) Let $W_1 = \{(x, y) \mid x, y \in \mathbb{R},\ y = 3x + 5\}$ and $W_2 = \{(x, y) \mid x, y \in \mathbb{R},\ y = 2x\}$. [6]

Show that W_1 is not a subspace and W_2 is a subspaces of $\mathbb{R}^2$ with usual vector addition and scalar multiplication.

(c) A Chemical Engineer is investigating the effect of process operating temperature x on product yield y. The study results in the following data, [8]

x :	100	110	120	130	140	150	160	170	180	190
y :	45	51	54	61	66	70	74	78	85	89

Find the equation of the least square lines which will enable us to predict
(i) yield on the basis of temperature (ii) temperature on the basis of yield.

Q3. (a) Find the Extremal of [6]

$$\int_0^1 yy' + (y'')^2 \, dx, \quad y(0) = 0,\ y'(0) = 1,\ y(1) = 2,\ y'(1) = 4.$$

(b) Three factories A, B, C produce 30%, 50% and 20 % of the total production of an item. Out of their production 80%, 50% and 10% are defective respectively. Find the probability of an item chosen at random is defective. If an item chosen is found to be defective, find the probability that it was produced by the factory B. [6]

(c) Find a singular value decomposition of the matrix $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$. [8]

Q4. (a) Evaluate the following integrals using Cauchy Residue theorem, [6]

$$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)^2} \, dz, \quad C: |z| = 3.$$

(b) Find the usual inner product between the two vectors, $(1,2,0,1)$ and $(-1,0,1,3)$. Find the norm of each vectors and verify the Cauchy Schwarz inequality. [6]

(c) The income group of 10,000 people were found to be normally distributed with mean Rs. 520 and standard deviation Rs. 60. Find the number of people having income (i) more than Rs 600, (ii) between Rs. 400 and 550, (iii) less than Rs 450. [8]

Q5. (a) Evaluate using Cauchy integral formula, [6]

$$\int_C \frac{(z+4)^2}{z^2(z^2+5z+6)} \, dz, \quad C: |z| = 1.$$

(b) Calculate the rank correlation coefficient for the following data. [6]

x	:	10	12	18	16	15	40
y	:	12	18	20	15	50	25

(c) Using Rayleigh-Ritz method, find an approximate solution for the extremal of $\int_0^1 2xy - y^2 - (y')^2 \, dx$, $y(0) = 0$, $y(1) = 0$. [8]

Q6. (a) Find the extremal of $\int_{x_1}^{x_2} \sqrt{1+(y')^2} \, dx$. [6]

(b) Find the Laurent series expansion of $\frac{2}{(z+1)(z+3)}$ convergent in the region i) $|z| < 1$ ii) $|z+1| > 2$. [6]

(c) Reduce the quadratic form $x^2 + 2y^2 + 2z^2 - 2xy - 2yz + xz$ to a diagonal form using congruent transformation and find its rank, index and class value. [8]